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CALCULATION OF FLOWS AROUND ZERO THICKNESS WINGS WITH EVOLUTIVE VORTEX SHEETS

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16. Abstract

The correct evaluation of the aerodynamic characteristics of some wing shapes requires the knowledge of the geometry of their vortex sheets. Out of the many calculations performed in this field at O.N.E.R.A., two are presented. The first one concerns a rectangular wing of very small aspect ratio (AR = 1), the second a sweptback wing of moderate aspect ratio (AR = 3.78). In both cases they are without thickness ("lifting surfaces") in incompressible flow. Their vortex sheets originate at the trailing edge and the wing tips.

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CALCULATION OF FLOWS AROUND ZERO THICKNESS

WINGS WITH EVOLUTIVE VORTEX SHEETS

Colmar Rehbach *

I. INTRODUCTION /54 **

The exact calculation of a perfect fluid around a lifting wing requires the knowledge of the geometry of its vortex sheet. The problem of determining the equilibrium position of the sheet has led to a number of publications in recent years. We will carry out this discussion within the framework of these articles. Here we will select two examples among many carried out in this area at the O.N.E.R.A. to show the great influence which the exact geometry of the sheet has on the evaluation of the aerodynamic characteristics of wings having certain shapes. Also we will show the good stability characteristics of the vortex shapes found, with respect to certain discretisation parameters. This will demonstrate the validity of the discretisation scheme used in order to establish the strict convergence of the method. This study only deals with wings having zero thickness (lifting surfaces) in an incompressible flow and with vortex sheets which originate at the trailing edge and at the wing tip. We do not consider the formation of a sheet at the leading edge.

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** Numbers in the margin indicate pagination of original foreign text.

II. FUNDAMENTALS OF THE NUMERICAL METHOD

It is known that the aerodynamic effect of an infinitesimally thin lifting surface and its sheet can be represented by a vortex layer. In the appendix we give the exact integral formulas which give the velocity induced by such a layer, as well as the forces which it applies to the surrounding fluids.

For the numerical calculation, this continuous analytic representation must be replaced by a system of discrete vortices. Here we will adapt a system of horseshoe vortices (see Figure 1). The final results as well as the organization of the calculation program will be simplified by doing this. An additional simplification consists of approximating each of these horseshoe vortices by a rectilinear segment which represents the so-called "bound" part, and a continuation to each side which consists of rectilinear segments of vortices called "free". The problem is to distribute the vortex system geometry over the wing. On the other hand, beyond the trailing edge and beyond the tips, the vortices will freely develop and will take on an equilibrium position defined by the condition of constant pressure through the sheets. Within the framework of the discretisation adapted in this study, this leads to the requirement that each of the vortex elements making up the sheet be parallel to the local velocity vector.

1. Iteration calculation of the sheet

The unknowns of the problem are the bound vortex intensities and the geometry of the sheet. The intensities are the solution of a system of linear equations obtained by imposing the sliding condition

$$\vec{V}.\vec{n} = 0,$$

at as many points on the wing (control points) as there are unknown intensities, where $\vec{V}$ is the vector velocity and $\vec{n}$ is the normal to the surface of the obstacle at the point under consideration. The velocity vector is made up of the upstream velocity and the velocity induced by the complete vortex system. Therefore, it is a function of the other unknown of the problem: the geometry of the sheet in the equilibrium position. This can only be solved by using an iteration procedure.

The paper by S. M. Belotserkovskiy [1] is one of the oldest on this subject. At the beginning, he only used his results. Because the calculation times became very long, we were led to combine this method with another procedure which is more rapid and which is described by D. J. Butter and G. J. Hancock [2]. The calculation time reductions obtained by doing this amount to 50%. The reason is as follows: Procedure [1] requires that there is a progressive increase in the incidence angle in order for the sheet to converge to the equilibrium position in a continuous way. When starting from a very small incidence, the incidence found during the calculation is only found after passing through several intermediary incidence values. An approximation of the equilibrium position of the sheet is calculated by iteration for each one of them. It can therefore be seen that the total number of iterations to be carried out for each calculation increases rapidly with incidence angle. An important improvement can be made if procedure [1] can be applied to a good approximation of the sheet at the calculated incidence. The method of Butter and Hancock [2] is what is used in the present version of the program. In the original article, it was restricted to lifting configurations without edge vortices. However, it is easily extended to a sheet which includes edge vortices.

We adapted the rules given by P. E. Rubbert [4] and S. G. Hedman [5] to determine the control point positions and the arrangement of the vortex grid with respect to the leading edge of the wing. According to R. M. James [3], this is important for the accuracy which can be obtained with this type of discretisation. These are the results of an analysis of very many numerical examples, which were completely confirmed by the theoretical work of R. M. James, already mentioned. This is especially true for reference [4].

2. Calculation of local and global aerodynamic effects

2.1 Calculation of the pressure coefficient from the induced velocity.

The pressure coefficient at a control point can be obtained from the velocity at this point. The sliding condition is satisfied and therefore this velocity is tangential to the wing surface. In order to evaluate it, we use a discretisation formula (3) given in the appendix.

The surface integral which represents the first term of the second expression in (3) can be calculated with a good degree of approximation from the discrete vortex system. The second term, on the other hand, which involves the local vortex intensity, disappears in the discretisation process. In order to find an approximate value, one must make a transformation from circulations concentrated at the vortices making up the edges of a four-sided network (see Figure 1) to an equivalent continuous distribution. In order to do this, our program contains a procedure which is described in detail in the article by P. E. Roberts [4].

By proceeding in this manner, we obtained an average value of the pressure at the control points. The sum of these average values gives a good approximation of the lift for adapted wings or wings which are close to adaptation (ΔKp zero or small at the leading edge). Also this gives a good approximation of the pitching moment and the induced drag. In the case where there is no adaptation, the behavior of ΔKp near the leading edge is according to $1/\sqrt{\delta}$ (δ distance between the point under consideration and the leading edge). In order to obtain high accuracy results for the lift and the pitching moment, it is sufficient to consider this local variation. On the other hand, without any adaptation it is not possible to carry out a correct evaluation of the induced drag. The method does not consider the suction effect which is produced for the case of a leading edge of a thin wing.

It should be noted that if the velocity is specified at each control point, up to a factor of proportionality, which has been determined by establishing a linear system, the calculation of the pressures by this procedure just described is very fast and is particularly adapted to the natural flow of the program.

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2.2 Calculation of the pressure coefficient from average local forces.

A second calculation method of the pressure can be obtained by writing formula (6) in the appendix in a discrete way. This is the summation of the forces applied by the fluid on each of the vortex segments of the lifting system over the wing. By dividing the contribution of the four edges of each rectangle by its surface, we obtain the local average pressure.

This method requires the calculation of the velocities induced not only at the control points but also at intermediate points each located along the segments which make up the network. The calculation times will therefore be much larger than for the method described in the preceding paragraph.

The global forces calculated on the basis of the formula (6) automatically imply the regular behavior of the pressure at the leading edge and the effect of suction which is applied in the case of a non-adapted wing.

For various applications, the two methods of calculation of the aerodynamic effects were applied simultaneously. We were able to verify that the results obtained for the lift and for the pitching moment always agreed well. As far as the induced drag in the presence of suction is concerned, it has a more theoretical meaning. During wind tunnel tests, it is very hard to reproduce (separation at the leading edge). The experimental results which are available usually correspond to values without suction (see paragraph III,1). In order to take this into account in the following numerical examples, we evaluated them according to the method given in II, 2.2.

III. EXAMPLES

The vortex system which replaces the wing and its sheet for the numerical study consists of M bands which each consist of N bound vortices. In order to characterize the system, it is sufficient to specify the parameters (M x N). The organization of the calculation program is such that the system defined in this way will consist of N vortices emerging from the wing tip and M vortices which emerge from the trailing edge. If we do not give a representation of the edge vortices, this amounts to M + 1 vortices emerging from the trailing edge.

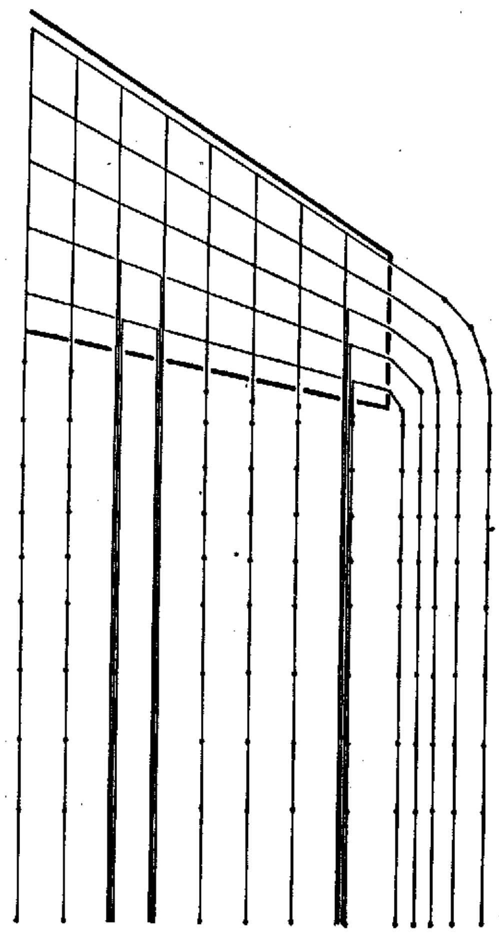

Figure 1. Discretisation of a lifting system with vortex sheet.

1. Flat rectangular wing

In this paragraph we will study a flat plate with a small aspect ratio $\Lambda = 1$. In the literature there are many theoretical studies as well as some experimental results on this type of wing. This will make it possible to verify whether the consideration of the equilibrium position of the complete vortex sheet will lead to results which are much better than those given by approximate methods used up to the present (linear theory of the lifting surface, Gersten method [6]) for calculating the global forces on wings with small aspect ratios. It is known that for wings, the lift, the induced drag and the moment will differ considerably from values obtained with the linear theory, even at moderate incidence angles.

Figure 2 shows the equilibrium position of the sheet obtained for a high incidence angle of 19.4°. The left side shows the entire vortex system projected on the plane $z/c = 0$. The right side shows traces of the sheet for various planes $x/c =$ const. From the sections $x/c = 0.5$ and $x/c = 1.0$, it is easily seen that the sheet leaves the wing tangentially, as is required by pressure continuity.

Figure 3 shows the experimental curves for lift, induced drag and pitching moment (around the leading edge) as a function of the incidence. This is compared with the theoretical results obtained by three different methods. The experimental values were obtained by N. Scholz [7] in the wind tunnel at the Technical University of Braunschweig. The dotted curves based on the linear theory of the lifting surface were provided by the same author. The dashed curves were taken from a publication by K. Gersten [6]. They take into account the non-linear effect of a sheet with edge vortices, which is represented by a rough model. The results of the present calculation are indicated by small squares. We would like to state that the vortex system was characterized by the parameter $(M \times N) = (5 \times 8)$ for all the incidence values and consisted of 5 vortices emanating from the trailing edge and 8 edge vortices.

As far as the lift is concerned, it can be seen that the linear theory gives low estimates even at small incidence angles. The Gersten method also provides values which always remain smaller than the experimental results. For the present method, we find that up to about 10°, there is good agreement between the theoretical and experimental values. Above 10° the experimental values are less than the calculated values. This could be due to the fact that according to N. Scholz, there is a separation with the formation of a vortex zone from the leading edge of a flat plate, already beginning with small incidence angle values. This has a profiling effect and considerably reduces the strong theoretical depression in this region. Figure 4 shows that the global value of the lift as well /56 its distribution over the span are well represented by our vortex scheme, at small and moderate incidence angle values. The figure shows the normal force coefficient along the span, which is easier to measure during an experiment. For the incidence angle of 7.8°, we also show the distribution according to the linear theory. Already for this small incidence angle, the non-linear effect primarily due to the rolled-up edge vortices is considerable. For large incidence angles, one found that the difference between the calculated C_N and the measured C_N is greatest in the center part of the wing. This seems to indicate that the separation along the leading edge occurs there.

As far as the pitching moment is concerned (Figure 3b), experiment and theory (O.N.E.R.A. program) agree up to an incidence value of about 16°. The linear theory and the Gersten scheme again result in values which are underestimated.

In the figure showing the induced drag as a function of incidence angle (3c) we show the results of the present method where we have taken the suction force into account along the leading edge. We also show the results when this force is /57 ignored. We can see that these values are very close to experimental results for the thin plate (separation at the leading edge, and therefore disappearance of the suction). The theoretical values with suction are only approached by wind

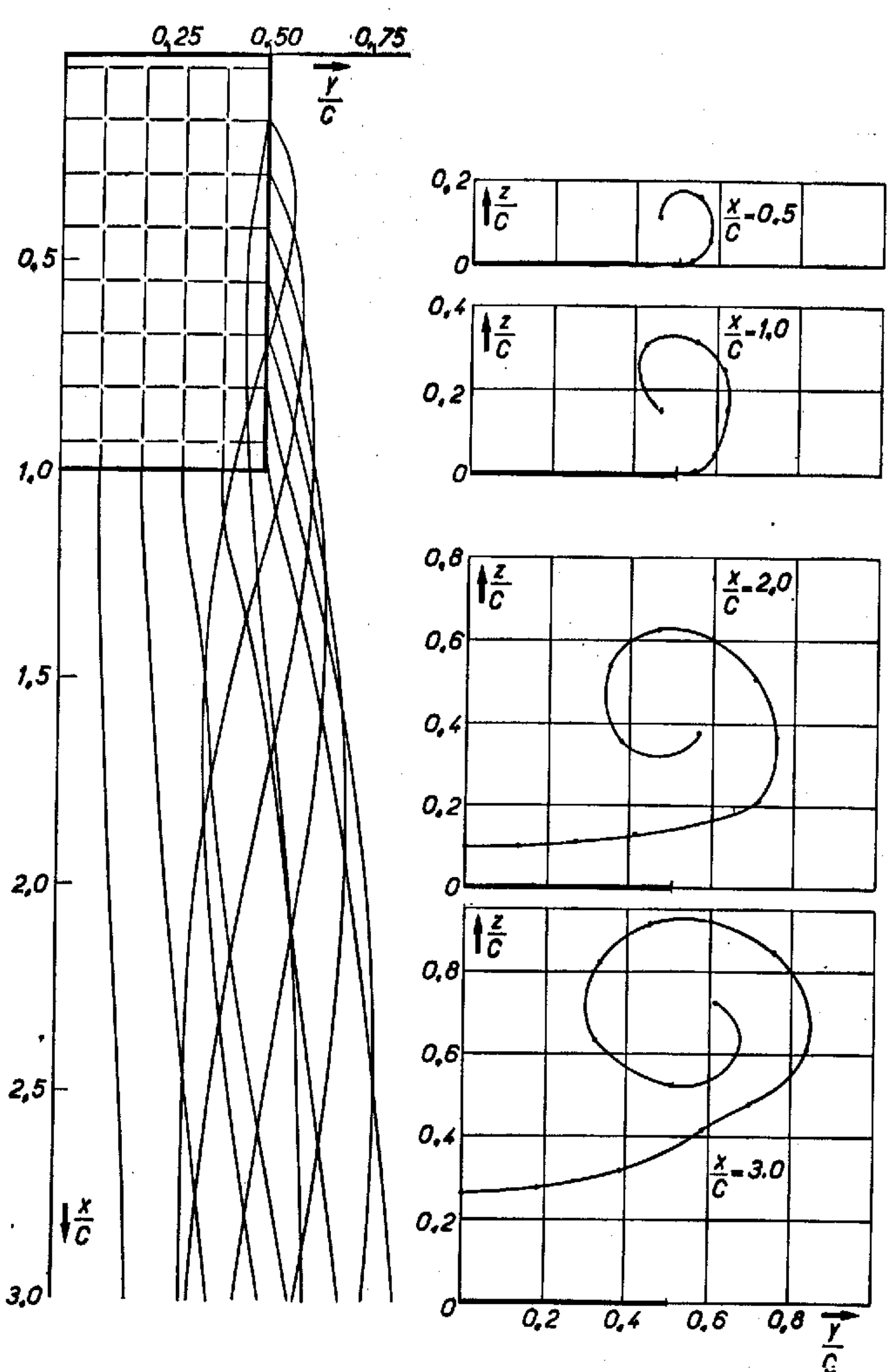

Figure 2. Rectangular plate with aspect ratio 1. Vortex sheet for $\alpha = 19.4°$.

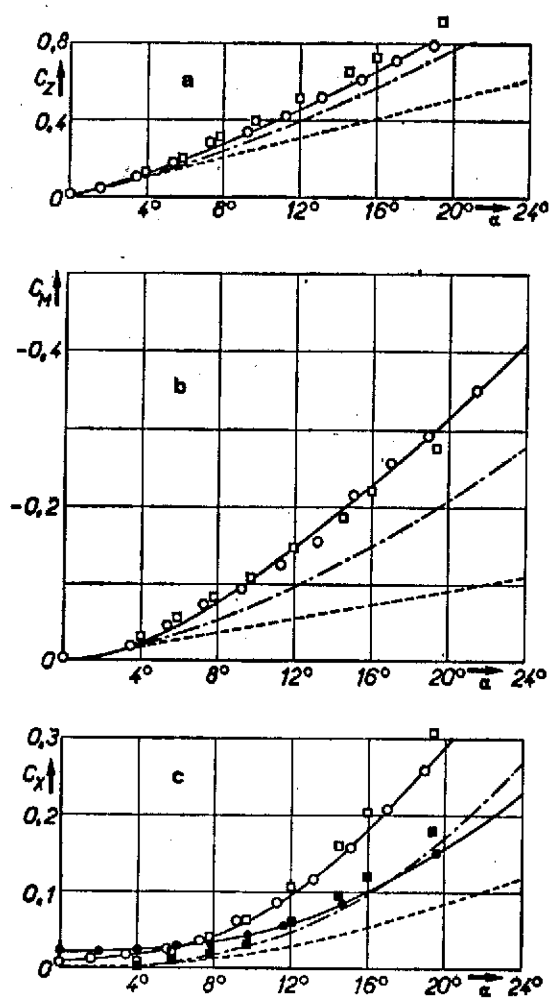

Figure 3. Rectangular plate with aspect ratio Λ= 1. Aerodynamic global values.

- ▫ O.N.E.R.A. calculations without suction,
- ▪ O.N.E.R.A. calculations with suction,
- ∘ Thin plate, experimental: N, Scholz [7],
- • Profiled plate Λ = 1.2, experiments: N, Scholz,
- —·— Gersten method [6]
- ----- Linear theory

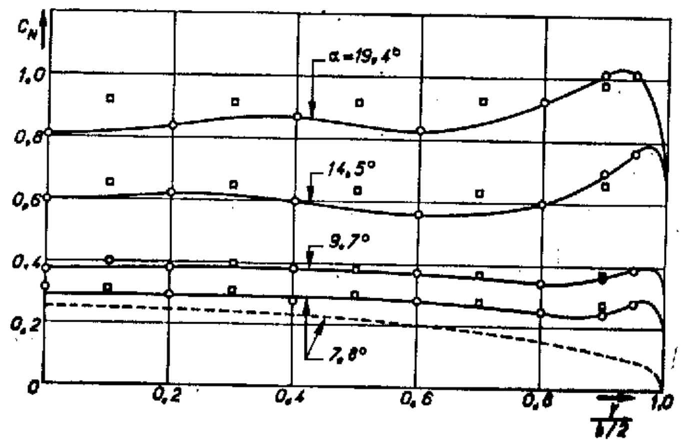

Figure 4. Rectangular plate with aspect ratio $\Lambda = 1$. Normal force over the span.

▫ O.N.E.R.A. calculation
∘ Experiment: N. Scholz [7]
····· Linear theory

tunnel experiments using a profiled plate.

By applying the reflection principle at the obstacle with respect to the plane $z = 0$, it is easy to consider the ground effect. For the flat plate studied here, such a calculation was carried out for an incidence angle of 16°. The distance between the trailing edge and the ground is $z/c = 0.27$. In the table below we show the lift, the induced drag and the pitching moment of the wing near the ground, divided by corresponding values in an infinitely large flow.

TABLE I

AERODYNAMIC GLOBAL VALUES WITH GROUND EFFECT

$\frac{C_Z}{C_{Zo}}$	$\frac{C_X}{C_{Xo}}$	$\frac{C_M}{C_{Mo}}$
1,203	1,220	1,240

In the article by G. H. Saunders [8] where the same wing is treated within the framework of the linear theory of the lifting surface, we find C_Z/C_{Zo} = 1.25 for the same distance from the ground. We also found the same tendency of a regression of the pressure center when one approaches the ground. Figure 5 shows a projection of the vortex system on to the plane y = 0 with the ground effect.

In order to show the order of magnitude of the calculation time required for determining the sheet by iteration as well as for the evaluation of the local and global aerodynamic values, we will now specify the times for a wing with an incidence angle of 9.7° (IBM 360-50):

Iteration calculation of the sheet: 31 min.
Calculation of aerodynamic parameters: 2 min.

We would also like to point out that the establishment of the equilibrium position of the sheet required 7 iterations (maximum displacement of the sheet from 1 iteration to the next less than 1% of the wing chord).

2. Swept back wing

We are dealing with a flat plate with a planform of a swept back wing. $(\vartheta_{b,a} = 30^0, \vartheta_{b,f} = 15{,}7^0$ with an aspect ratio of $\Lambda = 3.78$ (Figure 6). The rolling up of this sheet was already published in a short paper in the journal Recherche Aérospatiale (Aerospace Research) [9].

Here we will study the influence of the number of edge vortices on the shape of the rolled-up sheet and the effect on the global aerodynamic coefficients. To the calculation published in [9] whose vortex system was characterized by (M x N) = (5 x 6), here we will add the study of systems (M x N) = (5 x 8) and (M x N) = (5 x 10). A comparison study for an evolutive sheet will only include vortices emerging from the trailing edge (that is, without edge vortices). This was done as well as a calculation using the linear theory. The studied incidence angle was $\alpha = 8^0$.

Figure 6 shows the evolutive sheet for the case where the edge vortices are not represented. A comparison with the sheet with edge vortices (see Figure 7) shows that the representation of these edge vortices influences primarily the edge regions of the sheet. These are raised up more and are rolled up more. Figure 8 shows a summary of these conditions for the plane $\frac{x}{b/2} = 2.0$. We also show the influence of the /58
number N of edge vortices on the definitive shape of the sheet. The central vortex thread experiences a considerable displacement when a transition is made from 6 to 8 edge vortices. A further increase in N (from 8 - 10) produces a much smaller effect. This seems to indicate that the convergence as a function of N of the sheet towards its definitive shape is obtained very rapidly. The fact that for N = 8 we are very close to the

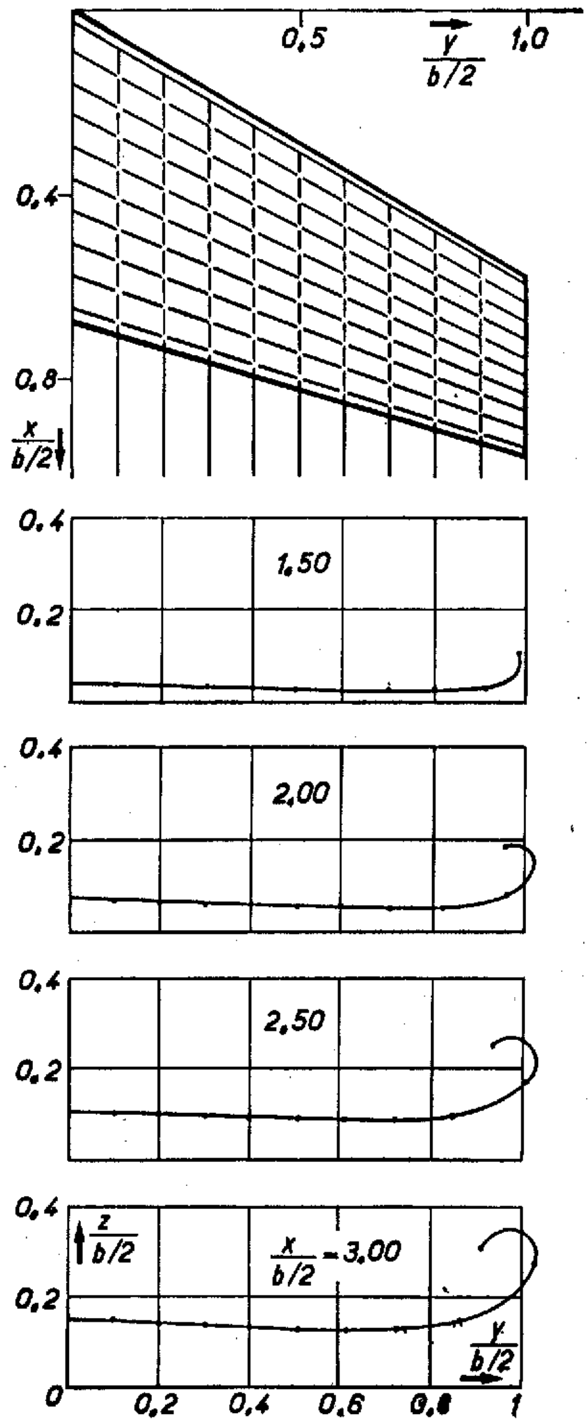

Figure 6. Swept back wing. Evolutive sheet without edge vortices for $\alpha = 8°$.

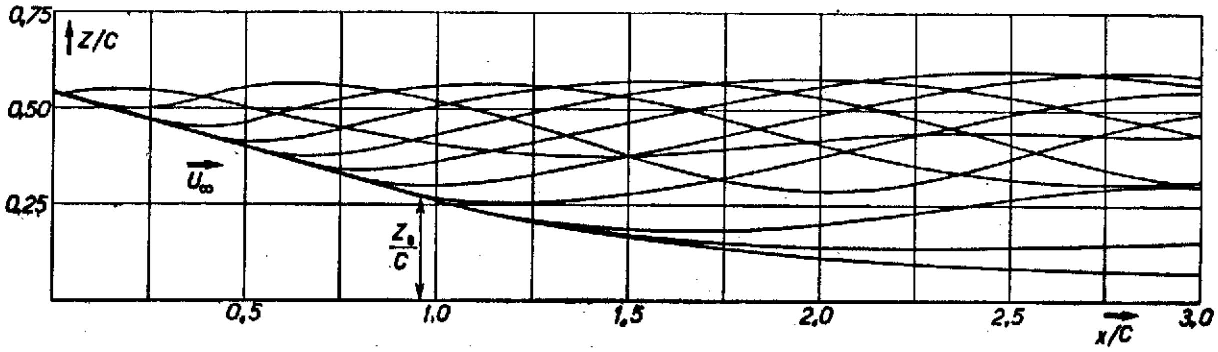

Figure 5. Rectangular plate with aspect ratio 1 in the presence of the ground.
$\alpha = 16°$, $Z_s/C = 0.27$

definitive shape was observed during numerous calculations and for a wide variety of lifting configurations. This is why we adapted this number during most of our studies of numerical applications of the program. We believe that this is an excellent compromise between the calculation time and reasonable numerical accuracy.

Figure 9 will show the influence of the discretisation mode of the sheet on the circulation distribution over the span. This is a wing with a medium aspect ratio ($\Lambda = 3.78$) and we would expect a moderate degree of influence. This has been confirmed by the calculation. It should be noted that in the case of the evolutive sheet without edge vortices, the circulation distribution on the scale of the drawing agrees with what is obtained using the linear theory. By taking the edge vortices into account, the circulation values are slightly increased, but their number only plays a subordinate role. In the drawing, the interval in which the values of the three configurations are located is equal to the height of the symbol used. It should also be noted that the values for 6 edge vortices are located at the upper limit and the values for 10 edge vortices are located at the lower limit of this interval.

The global aerodynamic values vary in the same way. In the table below we show the values of the linear theory.

TABLE II

INFLUENCE OF THE NUMBER OF EDGE THREADS ON THE GLOBAL AERODYNAMIC VALUES

	$\frac{C_Z}{C_{Zt.l}}$	$\frac{C_X}{C_{Xt.l}}$	$\frac{C_M}{C_{Mt.l}}$
(5 x 6)	1,051	1,042	1,102
(5 x 8)	1,047	1,040	1,078
(5 x 10)	1,045	1,037	1,061

The total calculation time (IBM 360-50) varies between 15 minutes for the (5 x 6) and 53 minutes for the system (5 x 10).

IV. CONCLUSION

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The calculation examples of the sheet presented in this note represent an example of numerous calculations carried out during the building of this program. Research was also carried out on how to use it most effectively. From our experience, we are able to draw the following conclusions:

- The geometry of the sheet depends to a slight degree on the number of edge vortices. The convergence of the sheet position as a function of this number is fast. Calculations assuming 8 edge threads result in good accuracy.

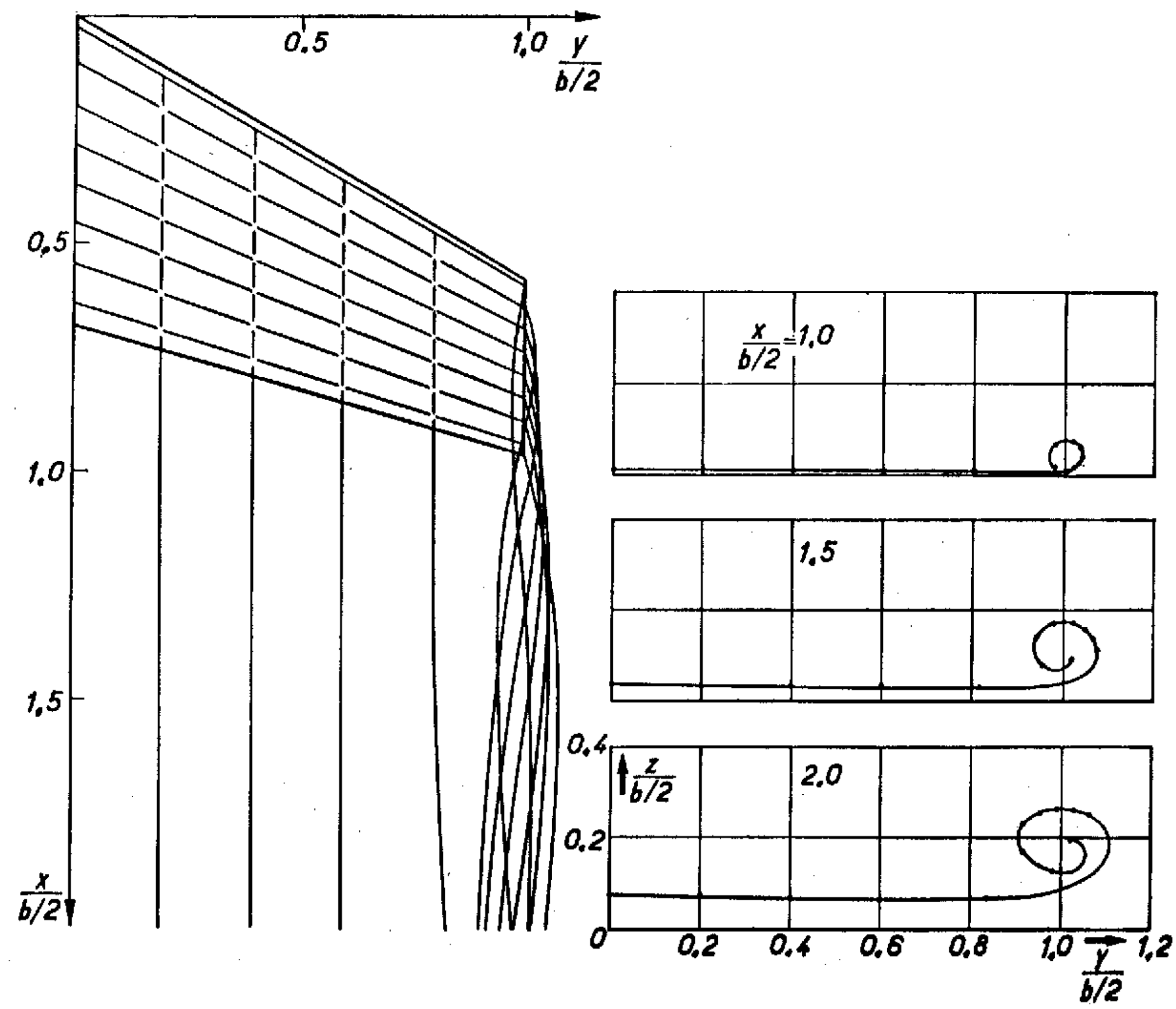

Figure 7. Swept back wing. Evolutive sheet with 8 edge vortices for $\alpha = 8°$.

- For low incidence angle values, the greatest number of problems are found in trying to calculate the equilibrium position of the sheet, as far as the numerical applications are concerned. These difficulties are primarily due to the fact that the edge threads are very close to each other at small incidence values.

- Knowledge of the shape of the sheet is indispensable for the correct evaluation of the aerodynamic parameters of small aspect ratio wings. When the aspect ratio is higher than 3, the

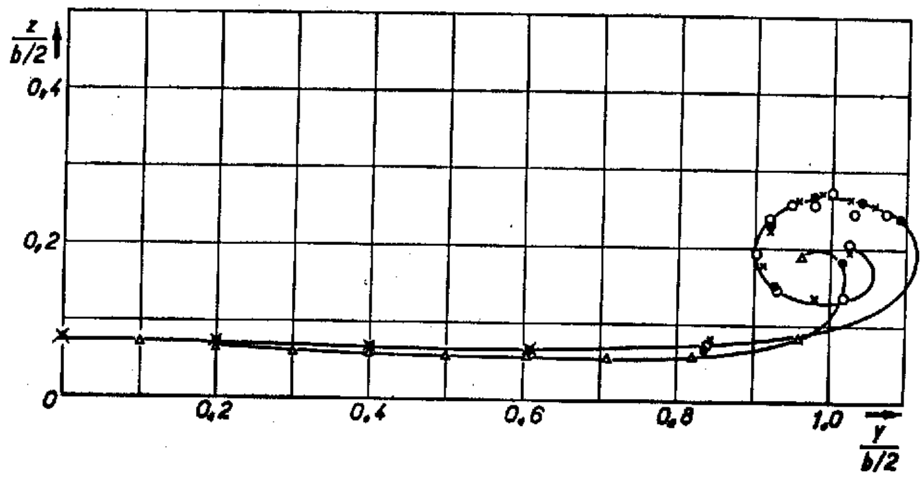

Figure 8. Swept back wing. Influence of the number of edge vortices on the shape of the sheet (plane $\frac{x}{b/2} = 2$).

▲ Without edge vortices
● 6 edge vortices
✱ 8 edge vortices
○ 10 edge vortices

sheet geometry becomes less important (at least up to moderate incidence values).

- The vortex scheme adopted seems to be satisfactory for lifting configurations which do not lead to separation at the leading edge. We are now studying an extension of the scheme to the case where there is a formation of a vortex sheet at the leading edge.

APPENDIX

The preceding numerical study is based on a discretisation of the exact formulas as follows [10]: /60

- A vortex distribution of density $\vec{\gamma}$ over a surface S

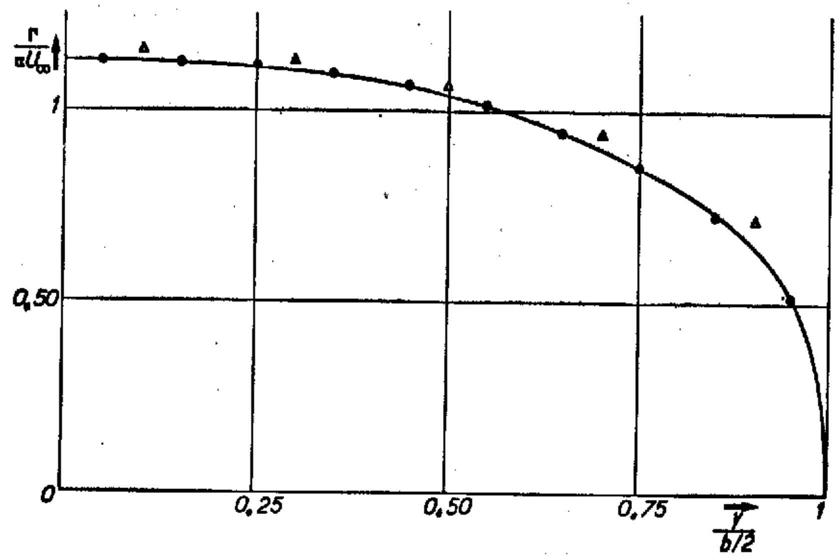

Figure 9. Swept back wing. Circulation over the span.

— Linear theory
o Evolutive sheet without edge vortices.
Δ { Evolutive sheet with 6 edge vortices.
Evolutive sheet with 8 edge vortices.
Evolutive sheet with 10 edge vortices. }

which induces the following velocity at any point P not located on S

$$\vec{v}_P = \frac{1}{4\pi} \iint_S \operatorname{grad}_P \left(\frac{1}{r}\right) \times \vec{\gamma}_Q \, ds_Q \qquad (1)$$

with $r = |\vec{PQ}|$.

When the point P is displaced and passes through the surface S, the velocity $\vec{v}$ is subjected to a discontinuity such that:

$$\Delta\vec{v} \equiv \vec{v}_{(+)} - \vec{v}_{(-)} = -\vec{n} \times \vec{\gamma} \qquad (2)$$

where $\vec{n}$ is the unit vector normal to S at P.

The velocity induced at a point P located on the side (+) or the side (-) of S is thus:

$$\vec{v}_{p\pm} = \frac{1}{4\pi}\iint_S \operatorname{grad}_p\left(\frac{1}{r}\right) \times \gamma_q ds_q \mp \frac{1}{2}\vec{n}_p \times \vec{\gamma}_p. \tag{3}$$

The second term in the integral is singular. The principal value in the sense of Cauchy must be taken. From (3) we derive the fact that the total velocity is obtained by adding the upstream velocity at infinity.

$$\vec{V}_{p\pm} = \vec{V}_\infty + \vec{v}_{p\pm} \tag{4}$$

Remark: The continuity condition for pressure through the vortex sheet provides for

$$|\vec{V}_{(+)}| = |\vec{V}_{(-)}|.$$

Considering relationship (2) which defines the orientation of $\Delta\vec{v}$ it follows that $\vec{\gamma}_p$ is parallel to the average velocity $\vec{V}_{pm}$ given by

$$\vec{V}_{pm} = \frac{1}{2}(\vec{V}_{p(+)} + \vec{V}_{p(-)}). \tag{5}$$

- The force which the fluid exerts on a vortex layer is:

$$\vec{R} = -\rho \iint_S \vec{\gamma}_p \times \vec{V}_{pm} ds_p. \tag{6}$$ /61

When there is pressure continuity through S, the vector product $\vec{\gamma}_p \times \vec{V}_{pm}$ is zero everywhere and we find the result about the direction of $\vec{\gamma}_p$ over the vortex sheet, already mentioned.

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